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Simultaneous determination of in- and over-tube heat transfer correlations in heat exchangers by global regression

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Abstract

We propose a method of data reduction that improves the predictions of correlations obtained from heat exchanger measurements. If we define an ideal heat exchanger on the basis of commonly made assumptions, the two heat transfer correlations corresponding to both sides of the heat transfer surface can be simultaneously determined. A local regression analysis, however, gives a multiplicity of possible correlations corresponding to the given data. The best correlations are obtained from this set by using a global regression procedure. Three methods are evaluated for this purpose: genetic algorithms, simulated annealing and interval analysis. All three perform well, with some differences in accuracy and CPU time. The predictions are further improved by correlating the error that is introduced by the assumptions of the ideal heat exchanger. The heat rate predictions are then improved considerably, giving a good idea of the extent to which these assumptions degrade them.

1. Introduction

Compact heat exchangers are common in industry, and for design purposes it is necessary to be able to predict their performance. This calculation is difficult from a first-principles perspective due to complexities arising from geometry, turbulence, and property dependence on temperatures. As a result, in practice most predictions are based on correlations that have been derived from experiments carried out on a specific heat exchanger [1]. Measurements are usually of flow rates and inlet and outlet temperatures of the fluids. From these the heat rate can be determined. Experimental data are used to obtain non-dimensional correlations which effectively compact the test results so that the user can employ these correlations to predict the heat rate of the same heat exchanger under different operating conditions and fluids.

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1.1. Ideal heat exchanger formulation

For the present discussion, consider a heat exchanger with two fluids A and B in single-phase flow, both geometry and fluids being considered fixed. To keep the heat rate non-negative we will assume that A and B are the hot and cold fluids respectively. We will refer to the analysis in this section as corresponding to an ideal heat exchanger. It basically reflects current practice and commonly made assumptions. The word *ideal* is used in the same sense as in ideal gas or ideal flow: the gas or the flow is real but the analysis has certain assumptions that enable simplifications.

The heat rate in a heat exchanger, $\dot{Q}$, is usually written as

$$\dot{Q} = U(\dot{m}_A, \dot{m}_B) A \Delta T_m (T_A^{in}, T_B^{in}, T_A^{out}, T_B^{out}) \quad (1)$$

where U is the overall heat transfer coefficient, $\dot{m}_A$ and $\dot{m}_B$ are the mass flow rates of the two fluids, A is a characteristic heat transfer area, ΔT_m is a characteristic temperature difference between the two fluids, T_A^{in} and T_B^{in} are the inlet temperatures of the two fluids, and T_A^{out} and T_B^{out} are their outlet temperatures (they are to be

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Nomenclature

A	heat transfer area (m^2)
A_r	area ratio
c	specific heat at constant pressure (J/kg K)
f	objective function
h	heat transfer coefficient (W/m^2 K)
j	Colburn j-factor
M	population in GA
m_A, m_B, n_A, n_B	correlation constants in Eq. (4)
$\dot{m}$	mass flow rate (kg/s)
N	number of experimental data sets
Nu	Nusselt number
n_b	number of bits in GA
n_1, n_2, n_3, n_4	correlation constants in Eq. (8)
Pr	Prandtl number
p_c	probability of crossover in GA
p_m	probability of mutation in GA
$\dot{Q}$	heat transfer rate between fluids (W)
$\dot{Q}_I$	ideal prediction of $\dot{Q}$ (W)
R	normalized Reynolds number
Re	Reynolds number
S	variance of error
$\overline{S}$	mean value of S for several runs
T	fluid bulk temperature (°C)
$\overline{T}$	fluid mean bulk temperature (°C)
T^r	reference temperature (°C)
ΔT_m	characteristic temperature difference (°C)
T^*	temperature-like parameter in SA
U	overall heat transfer coefficient (W/m^2 K)

Greek symbols

ϵ	error in ideal correlation
θ	normalized temperature
σ	standard deviation

Subscripts and superscripts

A, B	fluids A and B
e	experimental value
in	inlet
out	outlet
p	predicted value

interpreted as bulk temperatures). The equation is based on the entire heat exchanger being considered as a thermodynamic system, so that U and ΔT_m are not local quantities. ΔT_m in this equation must be defined for a given configuration in terms of the inlet and outlet temperatures, and is usually derived from analyses which have their own simplifications and assumptions [2].

The heat rate is also related to the inlet and outlet bulk temperatures by

$$\dot{Q} = \dot{m}_A c_A (T_A^{in} - T_A^{out}) = \dot{m}_B c_B (T_B^{out} - T_B^{in}) \tag{2}$$

where c_A and c_B are the specific heats of the two fluids at constant pressure. This relation is exact if the specific heats are constant (or an appropriate temperature-weighted average is used), and if there is no heat exchange other than between the fluids.

On invoking the concept of a thermal resistance, we can write

$$\frac{1}{UA} = \frac{1}{h_A(\dot{m}_A)A_A} + \frac{1}{h_B(\dot{m}_B)A_B} \tag{3}$$

where h_A and h_B are the convective heat transfer coefficients on their respective sides of the wall, and A_A and A_B are the corresponding heat transfer areas. The transverse thermal resistance of the wall due to conduction is usually negligible; conduction along it and between tubes has also been neglected [3]. Implicit in Eq. (3) is the assumption that the convective heat rate on one side of the tube wall is proportional to the difference between the bulk temperature of the fluid and the temperature of the wall. The thermal resistances on either side of the wall, $(h_A A_A)^{-1}$ and $(h_B A_B)^{-1}$, are thus assumed independent of the flow and temperature distributions on the *other* side. The sum in Eq. (3) comes from assuming that an identifiable, though not necessarily measurable, wall temperature exists. If all the quantities involved are assumed independent of position, then this equation is also valid in an overall sense.

From an experimental standpoint, Eq. (1) enables the total thermal resistance $(UA)^{-1}$ to be determined from mass flow rate and bulk temperature measurements. Eq. (3) tells us that, if the thermal resistance of one side is assumed or known, that of the other can be calculated. This procedure can be repeated for different flow rates. Assuming property values at a reference temperature in each stream, T_A^r and T_B^r, the heat transfer coefficients obtained can be converted to the non-dimensional form of a Nusselt number, Stanton number or Colburn j-factor. Finally, these can be correlated using regression analysis as a function of the corresponding Reynolds number, Prandtl number, or other non-dimensional parameters such as geometry that may have been varied. Thus only one correlation is obtained, that for the fluid on the other side being previously known in some manner or assumed. It must be emphasized that if the wall temperature could be directly measured, the thermal resistances and hence heat transfer coefficients on both sides could be independently determined from

experiments. However, in most heat exchanger configurations with internal and/or external fins it is difficult to find or specify a single location where the wall temperature should be measured.

1.2. Data analysis

Though other functional forms can be easily considered, let us for simplicity write power laws to correlate the Nusselt and Reynolds numbers on both sides of the heat exchanger to illustrate the data analysis process. Thus

$$Nu_A = n_A Re_A^{m_A} \quad \text{and} \quad Nu_B = n_B Re_B^{m_B}. \tag{4}$$

Other parameters like the fluid properties or geometrical factors are frequently included in the same way. The unknown constants are determined from experimental data using regression. The objective of the regression procedure is to find the values of the unknowns, like n_A, m_A, n_B and m_B in the equations above, that will minimize the variance of the error between the predictions of the correlations and the experimental data.

Wilson, in a pioneering paper in 1915 [4], proposed a graphical method to carry out a two-unknown regression analysis by reducing the equation to a straight line and then graphically determining its intercept and slope. The method has been extended to more variables by iterating on two at a time [5,6]. Though the Wilson plot method has been frequently used in the past [7–10], modern computational capabilities and numerical optimization techniques make it somewhat obsolete and we will not consider it any further.

Current practice in the processing of heat exchanger data can be grouped by the number of unknown constants that are determined from the regression analysis. In simple cases this can be done analytically, but in general the procedure is numerical.

- *Two unknowns*: There are many ways in which experiments can be designed or assumptions made such that the regression reduces to two unknowns. (i) If the thermal resistance on one side is negligible compared to that on the other side because of high heat transfer coefficients, either due to condensation or large flow rates of a liquid [1,11], we are left with just two unknowns in Eq. (4). Khartabil et al. [6] creatively used this method on data sets that were obtained from *different* experiments in which first one side then the other had a negligible thermal resistance to get correlations for both sides. (ii) Sometimes the thermal resistance on one side is assumed to be a constant which may either be known [12] or unknown [4,7–9]. (iii) Others have assumed that the thermal resistance on one side is variable but known through some pre-determined correlation [13,14].
- *Three unknowns*: If one of the unknowns in Eq. (4) is assumed known, regression can be carried out for the other three. This was graphically and iteratively done by Briggs and Young [5] by cyclically taking two unknowns at a time. Khartabil and Christensen [15], on the other hand, used numerical analysis to solve the three-unknown set of algebraic equations. Kayansayan [16] also had three unknowns, some of them being from geometrical parameters.
- *Four unknowns*: Gray and Webb [17] and Kim et al. [18] used numerical methods to find four constants, two of which were for geometrical parameters. The thermal resistance on only one side of the tube wall was included in the correlation.
- *Five unknowns*: Abu Madi et al. [19] had five unknowns with two for the flow and three for geometrical parameters. Again only one side was considered.

The methods outlined above have several drawbacks that prevent improvements in the accuracy of the predictions in spite of better measurement techniques: (i) Special experimental facilities with condensing or rapid flows are needed which may not be easily available to the manufacturer, or which may not be worth setting up since the device is not normally expected to be used in that fashion or with those fluids. (ii) Complete correlations for both sides of the heat exchanger are not simultaneously obtained under normal operating conditions when both thermal resistances are finite and unknown. (iii) There will be errors resulting from the ideal heat exchanger assumptions inherent in Eqs. (1) and (3).

These factors, in addition to the specific nature of the correlation functions assumed for regression and experimental data compression, may put a limitation on the improvement of predictions. It would thus be better if heat transfer correlations on both sides of the tube were determined simultaneously from experimental data obtained under normal conditions. This is what we propose to do. We will show that this will lead to the need for global, as opposed to local, regression, and we will apply common global optimization methods to experimental data. Finally, the effect of the ideal heat exchanger assumptions made in getting these correlations will be evaluated.

2. Experimental data

Data for the analyses carried out here are from the following two different sources.

(a) *Heat exchanger 1* (HX1): These are the results of tests on a single-phase, single-row plate-fin-tube heat exchanger carried out at the University of Notre Dame. The heat exchanger had a nominal size of 457 mm × 610 mm with a single-row circuit of 12 tubes connected

by bends. These data were collected as part of previous work in a test facility described in [20–22]. The fluids used were air in the over-tube side and hot water within the tubes. Two hundred and fifty nine tests with different air and water flow rates and water inlet temperatures were carried out. The measured variables were the mass flow rates, and the inlet and outlet temperatures of the two fluids.

From a heat balance for the transverse flow of fluid B over a long pipe through which A is flowing, we can show that

$$\dot{Q} = \dot{m}_A c_A \left(T_A^{in} - \overline{T_B}\right)\left[1 - \exp\left(-\frac{UA}{\dot{m}_A c_A}\right)\right] \tag{5}$$

where the assumptions of Section 1.1 have been used. If $\overline{T_B}$, the outside temperature, is approximated by $\overline{T_B} = T_B^{in}$, we get

$$\Delta T_m = \frac{T_A^{in} - T_A^{out}}{\ln\left[(T_A^{in} - T_B^{in})/(T_A^{out} - T_B^{in})\right]}. \tag{6}$$

This characteristic temperature difference was used in Eq. (1) to calculate U, with fluids A and B being water and air, respectively. Properties for each fluid were assumed to be at the average temperature between its inlet and outlet.

(b) *Heat exchanger 2* (HX2): This was a multirow multicolumn fin-tube heat exchanger with a nominal size of 127 mm × 305 mm for which considerable data were collected and reported by McQuiston [23]. Air was the over-tube and water the in-tube fluid. The fin spacings were also varied. The focus of the study was the air-side heat transfer, which was reported as j-factors, and therefore high Reynolds number turbulent flow was used in the water side to make the thermal resistance on that side small. Though the data included dropwise and film condensation, only the dry surface measurements will be considered here.

3. Local and global regression

The heat transfer coefficients, h_A and h_B, from Eq. (4) can be substituted into Eq. (3). To find the unknowns n_A, m_A, n_B and m_B from the experimental data, the variance between the experimental and predicted values of the total thermal resistance, $(U_i^e A)^{-1}$ and $(U_i^p A)^{-1}$, for $i = 1, \ldots, N$, must be minimized. This is equivalent to minimizing

$$S_U(n_A, m_A, n_B, m_B) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{U_i^e} - \frac{1}{U_i^p}\right)^2. \tag{7}$$

3.1. Local regression

To see why global rather than local regression is necessary, we apply the latter to the data from HX1. The proposed correlations are the first Eq. (4) multiplied by $Pr_A^{0.3}$ and the second by $Pr_B^{1/3}$. A gradient-based method [24] is used for minimization. The results show a multiplicity of local minima stemming from the non-linearity of S_U with respect to its arguments. Although some non-linear relations may be transformed into a linear form (e.g. by taking logarithms), the function S_U in Eq. (7) is not linearizable.

As an example, two of the several local minima found for n_A, m_A, n_B and m_B are 0.102, 0.591, 0.030, 0.787 and 0.091, 0.626, 0.092, 0.631, respectively. The variances for these minima are within 17.9% of each other. Though the heat rate predictions are similar, the thermal resistance predictions of the two correlations differ by 12.5% on the air side and 10–40% in the water side [21]. This indicates one of the difficulties associated with decoupling the thermal resistances on the two sides that is a consequence of the ideal heat exchanger assumptions. This also raises the question as to which of the local minima is the "correct" one. In other cases of regression, as will be shown later for HX2, a similar problem arises even when the correlation sought is only on one side of the tube wall. A possible answer is that we should use the correlation that provides the smallest value of the variance of the error over the entire range of available data, i.e. the global minimum.

3.2. Global regression

The search for the global minimum can be carried out by means of recently developed global optimization techniques such as genetic algorithms (GA), simulated annealing (SA), and interval methods (IMs). We briefly describe the methods here, though the details are in [25] and in the references cited. In our case the objective function f below for which a global extremum is being sought is a variance.

3.2.1. Genetic algorithms

GAs are stochastic, evolutionary algorithms that are based on the Darwinian principle of natural selection wherein the fittest members of a species survive and are favored to produce offspring [26]. The procedure is summarized as follows. A solution of four unknowns is encoded as a binary string of length n_b. An initial population of M strings is chosen and the objective function f is evaluated for each one of these. Pairs of parents are randomly selected but favoring those with the better values of f. Offspring are produced from these parents through the so-called processes of crossover (parts of binary strings switched between parents with probability

p_c) and mutation (random digit changed with probability p_m). Conserving the best member of the previous generation, a new population of M members is obtained. The process is repeated until some criterion based on convergence or the maximum number of generations is reached.

3.2.2. Simulated annealing

SA is inspired by the molecular calculation of the cooling of a physical system in which random agitation is used to avoid entrapment in local extrema [27]. A starting point in the space of unknowns is randomly selected on which a cycle of random moves along each coordinate direction is performed. The new point is accepted if it gives a better value of f. If it is worse, it is accepted only with a certain probability $\exp(-\Delta f/T^*)$, where Δf is the change in value of f and T^* is a dynamic parameter that is analogous to the temperature of a system being cooled [28]. The process is repeated with decreasing T^* and step size until convergence within a certain tolerance is reached. The procedure in [29] is followed here.

3.2.3. Interval methods

In contrast to the other two methods, the IM is a *deterministic* technique that is capable of finding the global extremum of an explicitly known f [30]. IMs use interval analysis which is based on an arithmetic of intervals. This enables some aspects of a function within a finite interval to be predicted just from calculations at its end points. A useful extension for optimization is the interval-Newton method. Given a non-linear system of equations with a finite number of roots within some interval, the method is capable of enclosing them between narrow bounds. The search for the global extremum of f begins with an interval which is divided into smaller intervals, each one of which is checked for the global extremum. If an interval includes the extremum, a new one is found using an interval Gauss elimination or a Gauss–Seidel-like technique. The procedure is repeated until the global extremum is bounded by a sufficiently narrow interval. Here we use a multidimensional extension [31] of the algorithm in [32].

4. Heat exchanger applications

To illustrate the global regression approach, we use data from HX1 and HX2. All the numerical computations were done using MATLAB so that the CPU times reported must be viewed in that context.

4.1. HX1

We first use GAs to find the four correlation constants of Eq. (4). Choosing $M = 30$ and $n_b = 30$ encodes them into a string of 120 bits providing a resolution of 1.4×10^{-9}. The search domain is chosen to be (0,1.5), with $p_c = 1$ and $p_m = 0.03$. Due to the stochastic nature of the method, slightly different results are obtained each time the code is run. An average value of the variance over 10 runs, $\overline{S_U}$, as well as its standard deviation, σ_U, are also determined. Three hundred and seventy three iterations were needed to achieve the global optimum. For SA, we take $T^* = 800$ initially, and the Kirkpatrick et al. [33] recommendation of a 10% decrease every time is used.

The errors obtained from the three methods are shown in Fig. 1 as a function of the heat rate. The same results are quantitatively summarized in Table 1. Shown are the values of the four constants n_A, m_A, n_B and m_B, the variance S_U, the CPU time for the best solution, the average $\overline{S_U}$ over 10 different runs, the rms of that σ_U, the rms percentage errors in U^{-1} and $\dot{Q}$ predicted by the correlations. It can be seen that all three methods provide values of S_U within 5% of each other, both IM and SA being equally accurate. On the other hand, the differences in $\overline{S_U}$ between GA- and SA-based correlations with respect to the IM-based correlation, which gives the least value, are 11% and 1% respectively. The reason for this is the larger variability of the GA every time it is run as compared to that of the SA. Since the IM is a deterministic method $\sigma_U = 0$. The results of the correlation developed by Zhao [22] using local regression and modified in [20], are also shown in the table for comparison. The global regression methods are much better with percentage errors confined to less than 3.8% and 3% in U^{-1} and $\dot{Q}$ respectively.

The IM is seen to give the best results. A comparison of $\dot{Q}$ between the correlation obtained from this method and experiments is shown in Fig. 2. The straight line represents the equality between the predictions and the experimental measurements.

4.2. HX2

A second demonstration of the global optimization analysis is with the data of HX2. The correlation proposed in [12] is

$$j = n_1 + n_2 Re^{-n_3} A_r^{-n_4} \tag{8}$$

where A_r is a geometrical parameter representing an area ratio, and n_1, n_2, n_3 and n_4 are constants. A slightly different correlation was reported by Gray and Webb [17] using data from [23] and other sources. The variance of the error between predictions and experiments of the j-factor is

$$S_j = \frac{1}{N}\sum_{i=1}^{N}\left(j_i^e - j_i^p\right)^2 \tag{9}$$

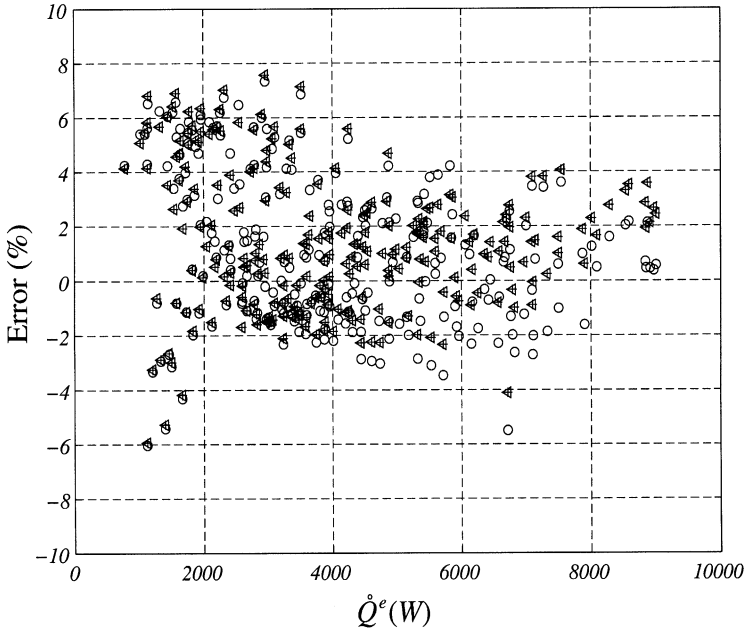

Fig. 1. Errors in heat rate predictions for HX1 with ideal correlations obtained by GA (○), SA (◁) and IM (+).

Table 1
Comparison of correlations for HX1

	$n_A \times 10^2$	m_A	n_B	m_B	$S_U \times 10^6$	CPU (h)	$\overline{S_U} \times 10^6$	$\sigma_U \times 10^9$	U^{-1} (%)	$\dot{Q}$ (%)
[20,22]	1.83	0.752	0.1368	0.585	–	–	–	–	21.4	9.16
GA	2.49	0.814	0.121	0.557	5.78	2	6.19	539	3.79	2.96
SA	3.03	0.785	0.102	0.592	5.49	5	5.49	3.07	3.64	2.87
IM	3.04	0.785	0.102	0.592	5.49	120	5.49	0.0	3.63	2.87

where j_i^e are the experimental measurements, and j_i^p are the predicted values, for $i = 1, \ldots, N$. It is found that S_j has multiple local minima [34]. The multiplicity of local minima of the error surface comes from the mathematical form of the correlating function assumed, Eq. (8), for which the non-linear function S_j is not linearizable, rather than from having to determine the correlations on both sides of the tube as we saw for HX1.

As before, we search for the least among the various local minima of S_j. The search domain is chosen to be $(-0.6, 0.6)$ for all the unknowns. We use first the GA choosing $M = 40$, $n_b = 30$, $p_c = 1$ and $p_m = 0.03$. The best set of unknowns is found after 2959 generations. In SA we take $T^* = 1500$ initially with a reduction of 15% every temperature adjustment. The tolerance is 10^{-9}. The number of cycles before temperature and step-change adjustments is 40. The number of iterations needed to achieve the global optimum is found to be 236. In the IM, the tolerance for enclosing the solutions is also set to 10^{-9}.

The error is shown in Fig. 3 as a function of the j-factor. The results are quantitatively summarized in Table 2. Again, the GA-based method is the most variable and the IM the most accurate. Fig. 4 shows comparisons of the j-factors between experiment and predictions using the IM. The table and this figure also show the results of [12,17]. All three global correlations are better than their published counterparts. Both IM- and SA-based correlations are almost equally accurate, with the GA-based correlation being very close.

5. Non-ideal corrections

So far we have considered ideal heat exchangers for which the assumptions in Section 1.1 hold. To summarize, the necessary and sufficient conditions for a heat exchanger analysis to be ideal are (i) the temperatures measured are the bulk temperatures; (ii) Newton's law of cooling applies and the resulting heat transfer coeffi-

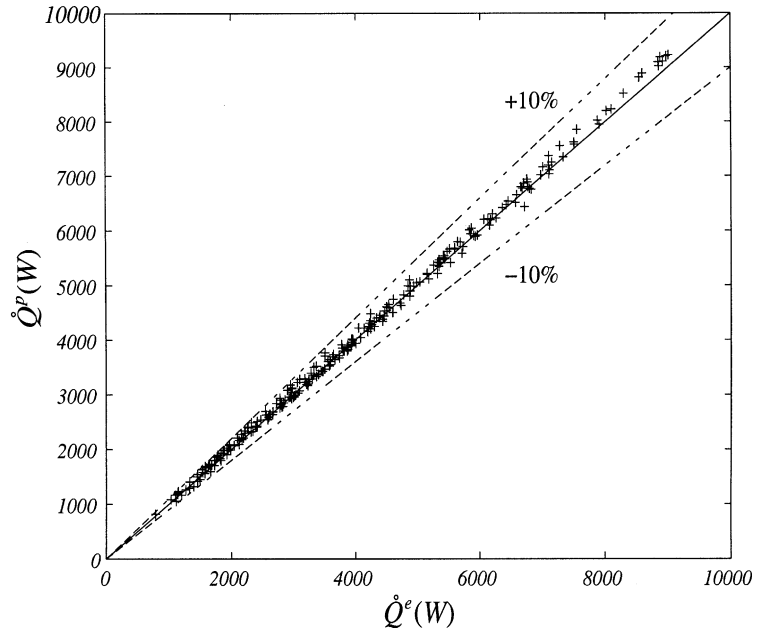

Fig. 2. Experimental vs. predicted heat rates for HX1 by the IM-based ideal correlation. Straight line is the perfect prediction.

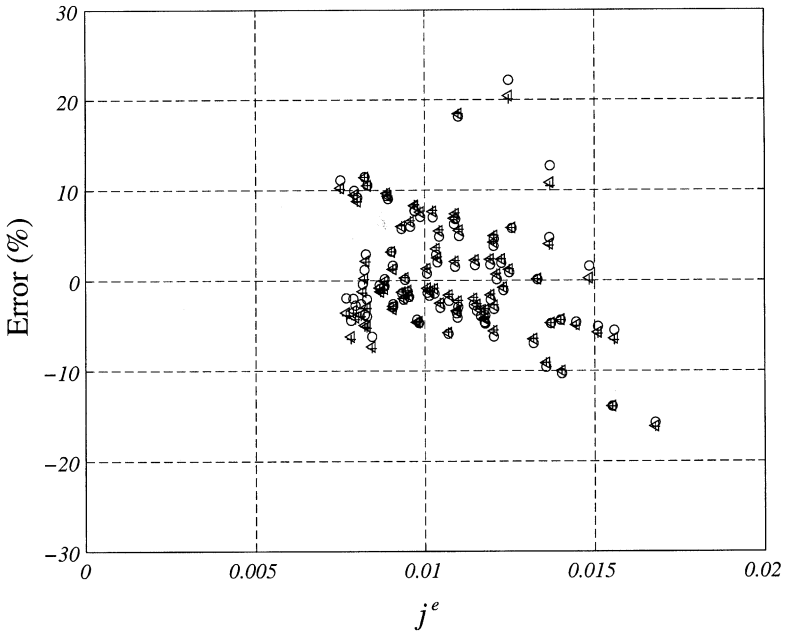

Fig. 3. Errors in heat rate predictions for HX2 with ideal correlations obtained by GA (○), SA (◁) and IM (+).

cients are independent of location; (iii) the heat transfer coefficients are independent of temperature differences and hence of each other; (iv) a measurable characteristic temperature difference between the two fluids that is independent of flow rates exists. These conditions are only approximate in real heat exchangers, largely because additional physics which may play an important role in the performance are not included. As a consequence we may have effects like local non-uniformities in the heat transfer process, flow maldistributions, temperature-dependent properties, and conduction and buoyancy effects. In this section we correlate the errors

Table 2
Comparison of correlations for HX2

	$n_1 \times 10^3$	$n_2 \times 10^5$	n_3	n_4	$S_j \times 10^7$	CPU (h)	$\overline{S_j} \times 10^7$	$\sigma_j \times 10^8$	j (%)	$\dot{Q}$ (%)
[12]	1.4	26,180	0.4	0.15	–	–	–	–	14.6	6.07
[17]	–	–	–	–	–	–	–	–	11.6	4.95
GA	17.9	−16.8	−0.456	−0.138	5.41	1.4	9.27	40.8	6.35	2.64
SA	16.9	−6.52	−0.552	−0.160	5.32	9	6.12	4.91	6.23	2.62
IM	16.7	−5.43	−0.570	−0.166	5.32	198	5.32	0.0	6.21	2.62

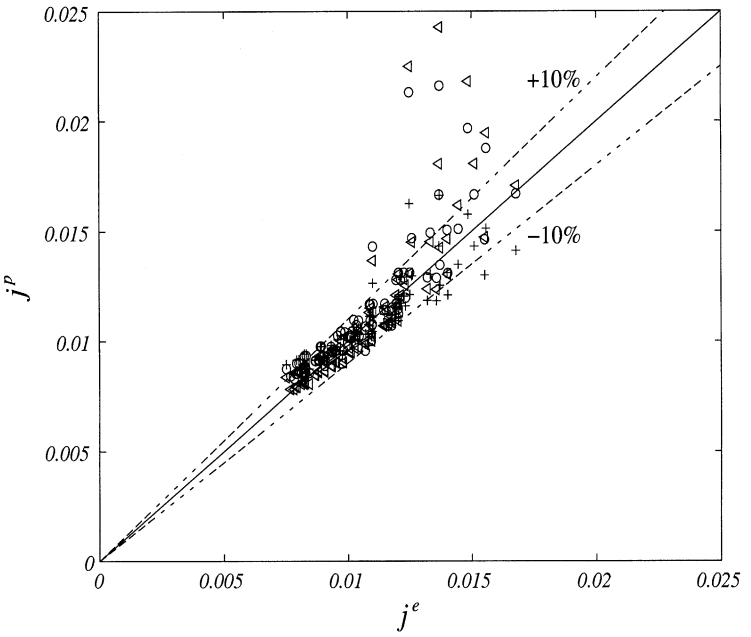

Fig. 4. Experimental vs. predicted heat transfer rates for HX2 by the IM-based ideal correlation (+). Also shown are the predictions of [12] (◁) and [17] (○). Straight line is the perfect prediction.

resulting from an ideal analysis to show that the physics missing is real, and is not simply a result of experimental inadequacies.

For this purpose, let us write

$$\dot{Q} = \dot{Q}_{\mathrm{I}}(1+\epsilon) \tag{10}$$

where $\dot{Q}_{\mathrm{I}}$ is the heat rate from an ideal analysis and

$$\epsilon = \epsilon(\dot{m}_{\mathrm{A}}, \dot{m}_{\mathrm{B}}, T_{\mathrm{A}}^{\mathrm{in}}, T_{\mathrm{B}}^{\mathrm{in}}) \tag{11}$$

Let us see if there is a pattern to ϵ or if it is completely random. To do this we find a correlation for ϵ as determined from the experimental data for HX1 and the corresponding prediction of the IM since that is the most accurate. The variance of the error in ϵ is

$$S_\epsilon = \frac{1}{N}\sum_{i=1}^{N}(\epsilon^{\mathrm{e}} - \epsilon^{\mathrm{p}})^2 \tag{12}$$

where ϵ^{e} and ϵ^{p} ($i = 1, \ldots, N$) are from experiments and predictions, respectively.

If ϵ were due only to random error, it would not correlate with the mass flow rates or temperatures. However

$$\epsilon^{\mathrm{p}} = -0.099 + 0.19\left[1 + \exp\left\{3.56 - \frac{2.92}{1+\exp\left(1-4.83R_{\mathrm{A}}+1.22R_{\mathrm{B}}+2.77\theta_{\mathrm{A}}^{\mathrm{in}}-12.72\theta_{\mathrm{B}}^{\mathrm{in}}\right)} - \frac{2.09}{1+\exp\left(5.3+8.12R_{\mathrm{A}}-13.19\theta_{\mathrm{A}}^{\mathrm{in}}+15.01\theta_{\mathrm{B}}^{\mathrm{in}}\right)} - \frac{5.29}{1+\exp\left(1.84R_{\mathrm{A}}+2.94\theta_{\mathrm{A}}^{\mathrm{in}}-0.58\theta_{\mathrm{B}}^{\mathrm{in}}\right)} + \frac{8.3}{1+\exp\left(1.74+1.4R_{\mathrm{A}}+8.74\theta_{\mathrm{A}}^{\mathrm{in}}-3.02\theta_{\mathrm{B}}^{\mathrm{in}}\right)}\right\}\right]^{-1} \tag{13}$$

works well. Though this was determined mostly by trial and error and a more compact correlation could possibly have been found, it shows that there is a pattern in ϵ. A bias error in the measurements themselves would, of course, behave like true values. The Reynolds number and temperature in the equation have been normalized by

$$R_A = \frac{Re_A - Re_A|_{min}}{Re_A|_{max} - Re_A|_{min}}, \qquad R_B = \frac{Re_B - Re_B|_{min}}{Re_B|_{max} - Re_B|_{min}} \tag{14}$$

$$\theta_A^{in} = \frac{T_A^{in} - T_A^{in}|_{min}}{T_A^{in}|_{max} - T_A^{in}|_{min}}, \qquad \theta_B^{in} = \frac{T_B^{in} - T_B^{in}|_{min}}{T_B^{in}|_{max} - T_B^{in}|_{min}} \tag{15}$$

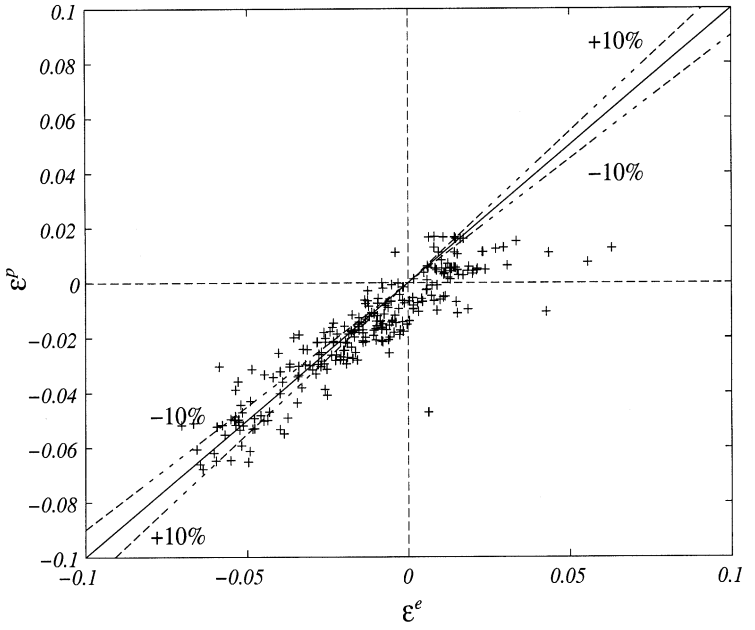

Fig. 5. Experimental vs. predicted ϵ for HX1. Straight line is the perfect prediction.

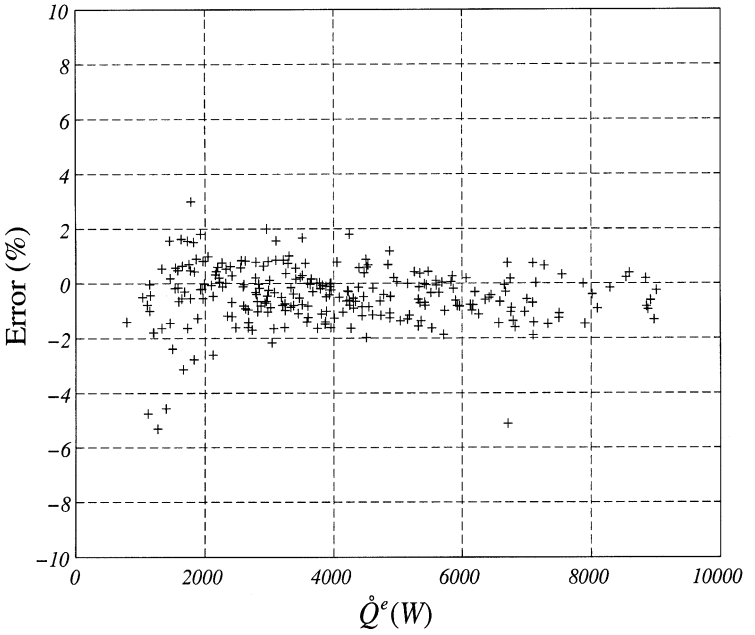

Fig. 6. Errors in heat rate predictions for HX1 including correction for non-ideal effects.

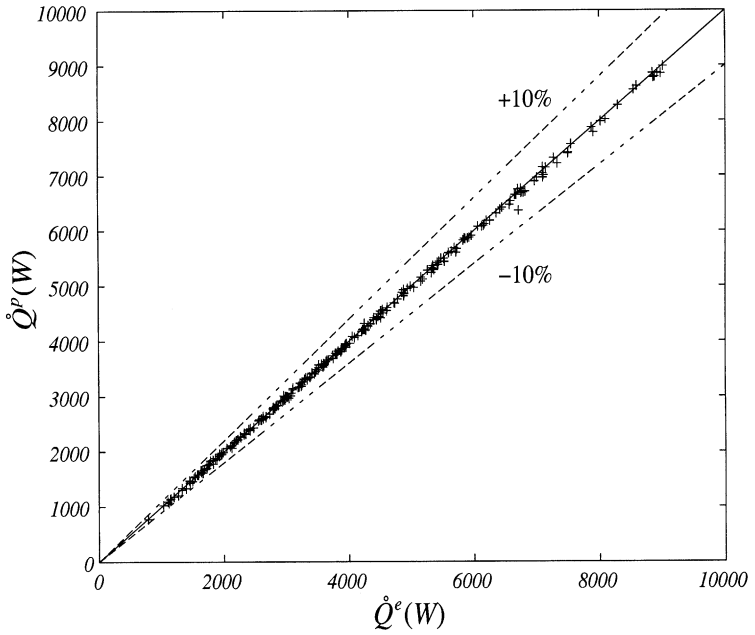

Fig. 7. Experimental vs. predicted heat rates for HX1 including correction for non-ideal effects. Straight line is the perfect prediction.

where the subscripts max and min stand for the maximum and minimum experimental values in the data sets for each one of the two fluids.

A comparison between the experimental ϵ and the prediction of Eq. (13) is shown in Fig. 5. The data are correlated quite well by the equation, indicating that the errors in the predictions from correlations derived using the ideal heat exchanger assumptions cannot all be attributed to random uncertainties in the experiments.

Now we use Eq. (13) in conjunction with the ideal correlation to make heat rate predictions that correct for non-ideal effects. Fig. 6 shows the new errors obtained this way. The rms error is 1.14% compared to 2.87% for the ideal analysis. Fig. 7 shows the experimental and predicted heat rates. The error in heat rate is 45.4 W compared to 102.7 W for the ideal correlations. Figs. 6 and 7 can be compared to Figs. 1 and 2, respectively.

6. Conclusions

The use of two heat transfer coefficients represented by non-dimensional correlations, one for the inside and another for the outside, has been a universally accepted vehicle for the characterization and prediction of heat exchanger performance. The simultaneous experimental determination of both these correlations, however, is not easy. We have proposed the concept of an ideal heat exchanger which leads to an analysis that is widely used but is based on a number of assumptions about the hydrodynamics, the temperature field and fluid property variations. The thermal resistances on each side can then be decoupled from each other. Further assumptions are also usually made to enable limited information about the correlations to be determined by regression from experimental data. Sometimes the regression leads to multiple local solutions. We have utilized a methodology based on global regression to avoid this problem. With this technique we can simultaneously determine the complete heat transfer correlations for both sides of the heat transfer surface from experimental data.

All three global techniques considered here, GA, SA and IM, are suitable for the purpose of finding, through regression analysis, the optimum values of the correlation constants. Each method has certain advantages over the others. GAs and SA do not require any gradient information to carry out the search and hence are faster than the IMs. GAs have an advantage over SA in being more efficient in the search as they produce a set of possible solutions each iteration. However, GAs are not guaranteed to obtain the global optimum but only the region in which it is located with high probability. SA, on the other hand, is probabilistically, though not deterministically, guaranteed to find the optimum. The major virtue of IMs is that, unlike GAs and SA, they guarantee that the global optimum has been found. Another advantage is that in the case of a multimodal function in which several minima exist, it is capable of finding all of those minima. However, the need for providing analytic expressions for the functions, gradients and Hessians are among the limitations of the

method. Another drawback is that in higher dimensions the algorithm is computationally very expensive.

The purpose of defining an ideal device is to enable the performance of a real machine to be compared to it. Since the assumptions made for an ideal heat exchanger hold approximately but not exactly, the behavior of a real heat exchanger is slightly different. If this non-ideal behavior is taken into account, the resulting error in predictions of the heat rates can be further reduced to about a third. This gives a good idea of the extent to which the ideal assumptions degrade the predictions and indicates what must be done to improve them.

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